

UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/investorsskewnes816sear>

7
p 2

R&D

BEER

FACULTY WORKING
PAPER NO. 816

Investors and Skewness Preference
in Option Portfolios

R. Stephen Sears
Gary L. Trennepohl

BY U. OF ILL. JUNE 1984

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

Set 112
Cap. 2

BEBR

FACULTY WORKING PAPER NO. 816

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

November 1981

Investors and Skewness Preference in Option Portfolios

R. Stephen Sears, Assistant Professor
Department of Finance

Gary L. Trennepohl
Arizona State University

DRAFT: Not to be quoted or reproduced in part or total
without written consent of the authors.

Abstract

This research is motivated by the belief that skewness in security returns is highly relevant to investors in long option portfolios. Because options possess positively skewed distributions, the traditional maxim of diversification, which can destroy positive skewness, may not be consistent with investment objectives. This paper presents theoretical justification for including skewness in evaluating option portfolios and presents analytical measures of portfolio skewness as a function of portfolio size. Our results indicate that the majority of skewness in option portfolios is diversified away at a relatively small portfolio size; however, the magnitude of nondiversifiable skewness is highly significant. Even though options are shown to perform poorly relative to stocks on a risk-return basis, their dominance once skewness is considered indicates the suitability of options in an environment where an investor's utility function is measured by the first three moments of the return distribution.

INVESTORS AND SKEWNESS PREFERENCE IN OPTION PORTFOLIOS

I. Introduction

The modification of a portfolio's return distribution which is made possible by call options requires that new thought be given to diversification in an option portfolio context. By making restrictive assumptions about investor utility, current financial theory explains security valuation using the first two moments of return distributions. However, the popularity of call options which have return distributions characterized by low means, high variances, but large positive skewnesses appears to be inconsistent with mean-variance analysis.

This research is motivated by the belief that skewness in security returns is highly relevant to investors in long option portfolios. Because options possess positively skewed distributions, the traditional maxim of diversification, which can destroy positive skewness, may not be consistent with investment objectives. Supporting this thesis is the observation that most retail option brokerage accounts which we surveyed typically held five or fewer options at any point in time.

This paper presents theoretical justification for including skewness in evaluating option portfolios and presents analytical measures of portfolio skewness as a function of portfolio size. Based on several years of security data, the behavior of option portfolio skewness measures for various stock price/exercise price ratios are examined and compared. Our results indicate that the majority of skewness in option portfolios is diversified away at a relatively small portfolio size; however, the magnitude of non-diversifiable skewness is highly significant. Even though options are shown to perform poorly relative to

stocks on a risk-return basis, their dominance once skewness is considered indicates the suitability of options in an environment where an investor's utility function is measured by the first three moments of the return distribution.

II. Options and Investor Utility

Theory relating investor utility to common stock is contained in previous literature [1, 4, 14, 16, 18, 28]. A brief review is necessary to relate the importance of option skewness to investor utility.

It is generally assumed that investors seek to maximize expected utility where utility is a function of investment return, R . Using a Taylor series expansion, expected utility can be expressed as a function of the moments about R :

$$E[u(R)] = u[E(R)] + \frac{u''[E(R)]}{2!} \sigma_R^2 + \frac{u'''[E(R)]}{3!} \sigma_R^3 + \dots + \frac{u^n[E(R)]}{n!} \sigma_R^n \quad (1)$$

where $E(r)$ is expected return, σ_R^2 is the variance about expected return, σ_R^3 is the skewness about expected return and σ_R^n is the n th moment about expected return. Even though the fourth and higher moments may play some part in explaining investor behavior, their importance is still unresolved. Our analysis will focus on the first three moments and their importance to option investors.

At least three reasons have been given for ignoring skewness when determining investor utility. First, if the utility function is quadratic, the third and higher moments are zero. Criticisms of the assumption of quadratic utility are contained in Jean [15] and Levy [17].

Second, if the return distribution is normal, the third moment will be zero. Previous studies of stock return distributions have provided ambiguous results which leaves the normality assumption open to question. Positively skewed stock return distributions are reported frequently in the literature [3, 4, 28], but Fama [10] indicates that distributions of continuously compounded rates of return sufficiently approximate normality. Still others [12, 15] have found the measurement of stock return skewness to be sensitive to the differencing interval as well as the sample period. Regarding options, however, research [20, 24, 29] consistently reports non-normal, positively skewed return distributions, thus supporting inclusion of the third moment in equation (1).

Third, some argue that if risks are small, the third as well as higher moments will be small and unimportant. Previous analyses of options [20, 24, 29] report large deviations in returns and much greater risk than their underlying securities. Omission of higher moments when evaluating utility from options may provide incorrect conclusions.

When evaluating the impact of skewness on asset pricing, it is necessary to consider together the first three moments of the return distribution. Generally, it is accepted that investors exhibit a preference for return, $u'[E(R)] > 0$, but an aversion to risk, $u''[E(R)] < 0$. Given these relationships, it can be shown [1] that $u'''[E(R)] > 0$, implying that investors have a preference for positive skewness and should be willing to accept a lower expected return from an investment having greater positive skewness, holding risk constant.

No empirical studies relating option returns to risk and skewness have been reported. However, given the nature of empirical option return

distributions, it seems apparent that rationale for option investment must lie with the skewness of returns. Inclusion of skewness in the investment decision necessarily complicates the process of utility maximization since investors also should be concerned with return and risk as well. In particular, since investors hold multi-asset portfolios, an understanding of the behavior of option skewness with increased portfolio holdings is necessary in establishing portfolio objectives.

The effects of diversification upon portfolio skewness can present the option investor with a set of complex tradeoffs. In the next section, we investigate the nature of these tradeoffs by analyzing the effects of diversification upon three important elements of option portfolio skewness.

III. An Analysis of the Components of Option Portfolio Skewness

Traditionally, security skewness has been measured on an ex-post basis by the skewness in the time series return distribution.¹ In a portfolio context, this skewness in return, σ_n^3 , on a portfolio of n securities is:

$$\sigma_n^3 = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n x_i x_j x_k \sigma_{ijk} \quad (2)$$

where x_i , x_j and x_k represent portfolio proportions invested in i , j and k and σ_{ijk} indicates the coskewness between the time series returns on i , j and k .

Under a policy of equal investment in each security, expected portfolio skewness is expressed as:

$$E(\sigma_n^3) = \left(\frac{1}{n}\right)^2(\bar{\sigma}^3 - \bar{\sigma}_{ijk}) + \bar{\sigma}_{ijk} \quad (3)$$

where $\bar{\sigma}^3$ is the average skewness for a one security portfolio and $\bar{\sigma}_{ijk}$ denotes the average security coskewness for the security population.²

Ex ante, it is difficult to predict the effect on $E(\sigma_n^3)$ of increasing portfolio size since the signs as well as the relative magnitudes of $\bar{\sigma}^3$ and $\bar{\sigma}_{ijk}$ are unique to each population. Thus, an empirical examination is necessary to measure the effects of portfolio size on the positive skewness present in option return distributions.

Often overlooked [4, 8, 23, 28] in the analysis of skewness is that the investor should not only be concerned with the mean portfolio skewness, $E(\sigma_n^3)$, but also with the dispersion about it. The greater the variability among portfolio skews at a given portfolio size, the greater the uncertainty concerning the skewness of the option portfolio actually chosen by the investor. Since returns vary considerably across options, omission of the variability consideration can result in a serious misstatement of the return and skewness expected by the investor.

Mathematically, this dispersion among portfolio skews is greatest when only one security is held and is zero when the market is held.³ Combining this consideration with the possible effects of diversification upon $E(\sigma_n^3)$ presents the option investor with a tradeoff between reducing the uncertainty about portfolio skewness vs. reducing the expected level of skewness.

Finally, it can be argued that there is a third skewness consideration facing option investors since for any portfolio smaller than the market there exists a probability that the average return on the portfolio will

differ from the return earned by the market. Put differently, there is a cross-sectional distribution of average returns. For options, this distribution should be highly skewed since the maximum loss is restricted, but the maximum return can be quite large due to the leverage involved. The presence of positive cross-sectional skewness implies that diversification can reduce the upside average return potential of the portfolio chosen. Cross-sectional skewness at any portfolio size is measured by equation (4):⁴

$$E(\bar{r}_n - \bar{r}_N)^3 = \left(\frac{1}{n}\right)^2 \left(1 - \frac{n^2 - 1}{N^2 - 1}\right) E(\bar{r}_1 - \bar{r}_N)^3 \quad (4)$$

where:

n = number of securities in the portfolio

N = number of securities in the market

$E(\bar{r}_n - \bar{r}_N)^3$ = skewness of expected returns on portfolios of size n

$E(\bar{r}_1 - \bar{r}_N)^3$ = skewness of the expected returns on the securities (\bar{r}_1) about the expected return on the market (\bar{r}_N)

One motivation for option investment is the opportunity for abnormally high returns from a small investment. Equation (4) indicates that the probability of selecting a portfolio with an abnormally high average return (relative to the market) is greatest when $n=1$ and disappears when $n=N$. The implications of this concept upon the diversification issue can best be illustrated with the following example. Suppose the population consists of only three options whose expected returns are -100%, -100% and 200%. Now suppose an investor randomly selects a portfolio comprised of these securities. Even though the average expected return is 0%, regardless if one, two or all three options are held, the upside return potential differs significantly across the three portfolio sizes. At

$n=1$, there is the opportunity to earn 200%, whereas when $n=2$ and $n=3$, the maximum return potentials are 50% and 0%, respectively. Given the small investment outlay, the option investor may be willing to trade some of the reduction in return uncertainty to preserve the upside return potential and thus be motivated to hold a portfolio smaller than the market.

In summary, the option investor who is motivated by a preference for positive skewness is faced with a complex set of tradeoffs when establishing an appropriate portfolio size. Unlike portfolio risk considerations [9], the tradeoffs do not proceed in a consistent direction. Having analyzed these relationships, we now empirically evaluate the implications of these results upon option performance.

IV. The Data and Methodology

The sample chosen includes the 136 stocks having listed options available on December 31, 1975. Securities not having complete price data on the Compustat tapes over the period July 1, 1963 to December 31, 1978, were eliminated, resulting in 102 sample securities for analysis. Although the choice of this particular stock group introduces a selection bias in the study, these securities represent over one-third of the population of listed option securities; thus, these results may be inferred to the current universe of optionable stocks.

Since listed options were not available until 1973, six month option premiums for the 102 stock sample were generated for the 15 1/2 year sample period using the Black and Scholes pricing model (5):

$$C = PN(D1) - Ke^{-rt}N(D2) \quad (5)$$

where: $D1 = [\ln(P/K) + (r + \frac{1}{2} v^2)t] / v\sqrt{t}$

$$D2 = D1 - v\sqrt{t}$$

The beginning of period price, P, was obtained from the Compustat tapes; time to maturity, t, was specified as 180 days; the daily equivalent of the six month commercial paper rate was proxied for the risk-free rate, r; and the variance rate, v^2 , was estimated from the log of daily price changes obtained from the CRSP tapes for the six months prior to each option pricing date. The impact of dividends on the option premium was considered by reducing the stock price by the present value of dividends paid during the life of the option (see [5]). The above data were used to generate option premiums, C, across three exercise prices (K): 10 percent in the money ($P/K=1.1$), at the money ($P/K=1.0$), and 10 percent out of the money ($P/K=.9$).

Use of Black-Scholes beginning of period option premiums is believed necessary to generate a sample period of sufficient length and to standardize the stock price/exercise price ratios. The similarity between Black-Scholes model prices and actual premiums previously has been demonstrated [20].

Semiannual returns (gross of commission costs) for each option for the thirty-one six month periods were calculated by dividing the call value at expiration by the beginning of period call value. Stock holding period returns include price appreciation plus dividends.

V. The Results

The objectives of the empirical analysis are threefold. First, after a brief review of the return distribution statistics, we will

examine the effects of diversification upon the positive skewness present in option return distributions. Second, an assessment is made of the implications of the variability in skewness and cross-sectional skewness statistics. Finally, portfolio skewness will be integrated with portfolio return and risk to evaluate the effects that increasing portfolio size has upon the return performances of option portfolios.

Average Distribution Statistics for Alternative Portfolios

Table I presents return distribution statistics for the four security groups examined. Line 1 reveals that average returns increase (.066 to .210) as one moves from stocks to options with successively higher prices, while total risk as measured by average security variance, σ^2 , increases from 6.969 to 759.347. That option portfolios contain a great degree of systematic risk is shown by either the market portfolio variance, σ_N^2 , or the average security covariance, $\bar{\sigma}_{ij}$ (lines 3 and 4).

Total skewness ($\bar{\sigma}^3$) and systematic skewness (σ_N^3 or $\bar{\sigma}_{ijk}$) data presented in lines 5, 6 and 7 show that all portfolios exhibit positive skewness and implies that increasing portfolio size will cause portfolio skewness to decline toward coskewness, $\bar{\sigma}_{ijk}$ (see equation (3)), for all security samples.

Cross-sectional variance, $E(\bar{r}_i - \bar{r}_N)^2$, line 8, and cross-sectional skewness, $E(\bar{r}_i - \bar{r}_N)^3$, line 9, are relatively small for stocks, but become progressively greater for options. For example, for out of the money options, the uncertainty about average return (line 8) is 23.024 which is over 100 times the size of the mean return of .210. Furthermore, the skewness in this return distribution (line 9) is highly significant, even after allowance for the dispersion in average returns.⁵ The large

Return Distribution Statistics for Alternative Stock and Option Samples*

	Symbol	Optionable Stocks	Options, P/K=1.1	Options, P/K=1.0	Options, P/K=.9
1. Sample mean return	$E(r_N)$.066	.158	.176	.210
2. Average security variance	σ^2	6.969	166.460	318.699	759.347
3. Market portfolio variance	σ_N^2	2.779	55.512	87.359	141.567
4. Average covariance	σ_{ij}	2.740	54.414	85.069	135.450
5. Average security skewness	σ^3	30.170	4,596.450	18,929.701	143,519.290
6. Market portfolio skewness	σ_N^3	5.471	415.573	901.862	2,313.971
7. Average coskewness	σ_{ijk}	5.470	415.533	900.913	2,302.398
8. Variance of mean security returns about sample mean return	$E(\bar{r}_i - \bar{r}_N)^2$.120	3.364	7.906	23.024
9. Skewness of mean security returns about sample mean returns	$E(\bar{r}_i - \bar{r}_N)^3$.040	3.490	14.157	167.595
10. Variance in variance	$E(\sigma_i^2 - \sigma^2)^2$.350	156.355	1,147.285	237,620.070
11. Variance in skewness	$E(\sigma_i^3 - \sigma^3)^2$	3.682	141,675.462	5,712,941.101	685,211,280.011

variance and skewness in option returns is attributable to a few contracts which showed average returns of several thousand percent, while the majority lost money.

Finally, lines 10 and 11 present evidence that there is also considerable dispersion among option risks and skews as well. Data in lines 8-11 serve as a caution to those attempting to assess the "typical" option.

Evaluation of the data in Table I illustrates a return-risk disadvantage for option portfolios. Even though average returns increase by approximately threefold from stocks to out of the money options (.066 vs .210) average systematic risk increases 50 times (2.779 vs 141.567). The fact that average systematic skewness increases in excess of 420 times (5.471 vs 2,313.971) is consistent with the hypothesis that investors are willing to sacrifice expected return for positive skewness in option portfolios. Since investors diversify their holdings to modify portfolio return distribution characteristics, it is instructive to examine the behavior of the elements of option skewness in response to changes in portfolio size.

Diversification and the Components of Skewness

Using the skewness and coskewness data for each group in equation (3), the mean values of the time series elements of portfolio skewness are presented in Table II. As noted earlier, since $\bar{\sigma}^3 > \bar{\sigma}_{ijk}$, diversification within this sample will reduce portfolio skew. Consequently, 96% of diversifiable skewness is destroyed with a five-security portfolio and 99% is diversified with a ten-security portfolio, regardless of the stock price/exercise price ratio considered. The rate at which

Table II

Diversification and its Effects upon the Time Series Element of
Skewness of Alternative Stock and Option Samples

Portfolio Size	Stocks*	Options,* P/K=1.1	Options,* P/K=1.0	Options,* P/K=.9	% of Diversifiable** Skewness Destroyed
1	30.17	4,596.45	18,929.70	143,519.29	—
2	11.65	1,460.76	5,408.11	35,304.22	75.0
3	8.21	880.08	2,904.11	17,993.16	88.9
4	7.01	676.84	2,027.71	11,128.45	93.8
5	6.46	582.77	1,622.06	7,951.07	96.0
10	5.72	457.34	1,081.20	3,714.56	99.0
20	5.53	425.98	945.98	2,655.43	99.8
30	5.50	420.18	920.94	2,459.30	99.9
40	5.48	418.14	912.18	2,390.65	99.9
50	5.48	417.20	908.12	2,358.88	99.9
102 (σ_N^3)	5.47	415.93	902.64	2,315.96	99.9
Minimum ($\bar{\sigma}_{ijk}$)	5.47	415.53	900.91	2,302.39	100.0
$\sigma_N^3/\bar{\sigma}^3$.18	.09	.05	.02	

*Calculated using equation (3) and lines 5 and 7 of Table I.

** = $\frac{\bar{\sigma}^3 - E(\sigma_n^3)}{\bar{\sigma}^3 - \bar{\sigma}_{ijk}^3}$, any sample's % may differ slightly due to rounding.

total skewness approaches its minimum level is the same across all strategies; however, the proportion of skewness which is destroyed via diversification $(1 - \sigma_N^3/\sigma^3)$ differs across the groups. The ratio σ_N^3/σ^3 (last line, Table II) indicates that a substantial difference exists in the correlation structures internal to the various security groups. Correlation is greatest for stock returns and systematically declines as the stock price/exercise price ratio declines. These results reveal that an attempt to reduce portfolio risk via diversification will also destroy portfolio skewness. Furthermore, this process is especially damaging in the high-risk, highly skewed strategies--a result which may reduce the utility from option investment.

The cross-sectional skewness, equation (4), is examined in Table III. The data reveal that this element is positive for all four security samples and dramatically increases as P/K declines. As seen in Table III, increasing portfolio size rapidly reduces the positive skewness inherent in the distribution of portfolio average returns. As previously discussed the importance of this component of skewness can be gauged in terms of how the diversification process affects investor return opportunities. Since a considerable amount of uncertainty exists concerning the mean return of the security chosen (Table I, line 8), there is motivation to diversify security holdings. Diversification, however, removes the right tail of the portfolio mean return distribution, at a rate faster than return uncertainty.⁶ Thus, choosing an appropriate portfolio size involves a tradeoff between decreasing return uncertainty as portfolio size increases, while reducing the probability of abnormally high returns.

Table III

Diversification and its Effects upon the Cross-Sectional Component
of Expected Skewness for Alternative Stock and Option Samples

Portfolio Size	Stocks*	Options,* P/K=1.1	Options,* P/K=1.0	Options,* P/K=.9	% of Diversifiable Skewness Eliminated
1	.040	3.490	14.157	167.590	---
2	.010	.872	3.536	41.885	75.0
3	.004	.388	1.571	18.607	88.9
4	.003	.218	.883	10.459	93.8
5	.002	.139	.565	6.688	96.0
10	.001	.035	.140	1.660	99.0
20	.000	.008	.034	.403	99.8
30	.000	.004	.014	.170	99.9
40	.000	.002	.007	.089	99.9
50	.000	.001	.004	.051	99.9
102	---	---	---	---	100.0

*Calculated using equation (4) and line 9 of Table I.

$$** = \frac{[1 - (\frac{1}{n})^2 (1 - \frac{n^2 - 1}{N^2 - 1})] E(\bar{r}_i - \bar{r}_N)^3}{E(\bar{r}_i - \bar{r}_N)^3}, \text{ any sample's \% may differ slightly due to rounding.}$$

Finally, the impact that diversification has upon the variance in skewness is illustrated in Table IV. Since this statistic equals line 11 in Table I when $n=1$ and is zero when $n=N$, our purpose is to examine the magnitude and behavior of this element as portfolio size is increased. To do this, we randomly selected, with replacement, 1000 portfolios of size 2, ..., 5, 10, 20, ..., 50. In each simulation, the same stream of random numbers was generated to facilitate comparisons.

As shown in Table IV, a small amount of diversification quickly reduces the uncertainty surrounding skewness. A portfolio of five securities has reduced more than 98% of the variance in skewness for stocks and more than 99% for option portfolios. But, due to the differences in magnitudes of the numbers, percentage comparisons can be misleading. For example, even though over 99% of the variance in skewness has been eliminated with five out of the money options, the magnitude of this uncertainty is more than 100 times the size of the mean level of skewness ($818,220.46/7951.07 = 102.9$). On the other hand, at the five security, this same comparison for stocks is only .008 ($.05/6.46 = .008$). On the whole, diversification can be detrimental due to its effects on the time series and cross-sectional elements, but beneficial in the reduction of skewness uncertainty.

VI. The Performance of Option Portfolios

Our last consideration concerns the effects that increased portfolio size has upon the investment performance of option portfolios. Traditional evaluation of performance uses the first two moments of the time series return distribution-- \bar{r}_1/σ_1 , or return per unit of risk. For securities possessing skewed times series return distributions, a second value--

Table IV

Diversification and its Effects upon the Variance in Skewness of Alternative Stock and Option Samples*

Portfolio Size	Stocks*	% of Variance** in Skewness Eliminated	Options,* P/K=1.1	% of Variance** in Skewness Eliminated	Options,* P/K=1.0	% of Variance** in Skewness Eliminated	Options,* P/K=.9	% of Variance** in Skewness Eliminated
1	3.68	---	141,675.46	---	5,712,941.10	---	685,211,280.01	---
2	.43	88.3	7,508.93	94.7	216,381.81	96.2	21,439,359.40	96.9
3	.15	95.9	3,694.54	97.4	123,686.87	97.8	12,524,753.91	98.2
4	.10	97.3	1,597.79	98.9	38,225.97	99.3	2,826,073.00	99.6
5	.05	98.6	590.61	99.6	11,315.78	99.8	818,220.46	99.9
10	.02	99.5	232.60	99.8	3,920.07	99.9	230,699.11	99.9
20	.01	99.7	64.02	99.9	678.71	99.9	26,351.27	99.9
30	.00	99.9	34.20	99.9	305.62	99.9	10,302.00	99.9
40	.00	99.9	24.43	99.9	200.55	99.9	5,668.72	99.9
50	.00	99.9	19.62	99.9	158.77	99.9	4,580.91	99.9
102	---	100.0	---	100.0	---	100.0	---	100.0

*Any sample's % may differ slightly due to rounding.

$$** = \frac{E(\sigma_1^3 - \sigma^3)^2 - E(\sigma_{1,n}^3 - \sigma_n^3)^2}{E(\sigma_1^3 - \sigma^3)^2}, \text{ any sample's \% may differ slightly due to rounding.}$$

$\sigma_i^3/(\sigma_i)^3$ ---skewness per unit of risk (or normalized skewness) can provide additional information concerning investment desirability [1]. Increasing values of normalized skew imply a benefit from diversification since the reduction in risk is not offset totally by the reduction in positive skewness. Dominant security positions are those investments which possess greater values of both measures of performance.

To investigate the impact of diversification upon option portfolio performance, we randomly selected, with replacement, 1000 portfolios of size 2,...,5,10,20,...,50. For each portfolio, the above time series measures were computed and then averaged to provide the $E[\bar{r}_i/\sigma_i]$ and $E[\sigma_i^3/(\sigma_i)^3]$ at each portfolio level. The size one and 102 values for these two measures were computed directly from the sample data. The results are presented in Table V.

Of course, $E[\bar{r}_i/\sigma_i]$ is everywhere increasing with diversification since portfolio expected return is constant but portfolio risk declines as n increases. Stocks dominate options implying that the additional risk from options is not adequately compensated by additional return.

The skewness/risk measure, however, shows options to be more desirable than stocks. The dominant strategy is out of the money options with the other option categories performing well vis a vis stocks at smaller portfolio sizes. Noteworthy is the divergent behavior of the normalized skewness measure. For the sample of stocks, skewness per unit of risk increases with diversification. However, for options this measure declines implying that reductions in risk are more than offset by losses in skewness as portfolio size is increased.

Time Series
Portfolio Performance Measures

Portfolio Size	$\frac{\text{Stocks}_3}{E[\bar{r}/\sigma] \quad E[\sigma^3/(\sigma)^3]}$	$\frac{\text{Options, } P/K=1.13}{E[\bar{r}/\sigma] \quad E[\sigma^3/(\sigma)^3]}$	$\frac{\text{Options, } P/K=1.9}{E[\bar{r}/\sigma] \quad E[\sigma^3/(\sigma)^3]}$	$\frac{\text{Options, } P/K=.9}{E[\bar{r}/\sigma] \quad E[\sigma^3/(\sigma)^3]}$				
1	.267	.829	.105	1.184	.070	1.623	.031	2.286
2	.317	.899	.142	1.057	.106	1.358	.067	1.873
3	.334	.986	.154	1.066	.121	1.323	.089	1.774
4	.348	.998	.171	1.060	.140	1.308	.109	1.732
5	.356	1.028	.174	1.031	.143	1.250	.114	1.650
10	.377	1.068	.194	1.018	.166	1.181	.144	1.520
20	.384	1.129	.202	1.016	.176	1.167	.156	1.498
30	.388	1.145	.205	1.016	.179	1.149	.163	1.462
40	.390	1.172	.208	1.024	.182	1.145	.167	1.438
50	.391	1.173	.208	1.018	.184	1.140	.170	1.435
102	.396	1.181	.212	1.005	.188	1.105	.176	1.374

VII. Summary and Conclusions

This paper has examined both theoretically and empirically the importance of skewness to holders of long option positions. Justification for including the third moment when measuring option investor utility is based on the arguments that (1) investor utility functions may not be quadratic, (2) option return distributions exhibit large positive skew and (3) relatively high risk exists in long option portfolios.

Three components of option skewness were presented and analyzed: time series, cross-sectional and the variance in skewness. Because diversification affects each component differently, the investor faces a complex set of tradeoffs when determining an optimal portfolio size. For option portfolios, diversification reduces time-series and cross-sectional skewness which is undesirable, while at the same time removes uncertainty about portfolio skew. Maximization of investor utility requires simultaneous consideration of the three components of skewness along with portfolio return and risk.

Because of the considerable uncertainty regarding the parameters of return, risk and skewness, an analysis of diversification can not determine an optimal portfolio size, but can only demonstrate the tradeoffs. Recent work [8] has demonstrated an optimization algorithm incorporating return, risk and skewness for a universe of homogenous assets; but, application of this technique to option portfolios appears impractical due to the heterogeneous nature of these securities.

Since the diversification-skewness issue entails several tradeoffs, it appears that a policy of some but not total diversification is optimal for option investors. This conclusion is supported by the desirability

of potentially skewed returns with small portfolios of options and the observation that option investors contain retail rokerage accounts of small undiversified portfolios.

Footnotes

¹In this paper we will use the term "skewness" to refer to a distribution's raw third moment. We will use the phrase "normalized skewness" to denote the raw third moment divided by the cube of the standard deviation.

²An equal investment policy is optimal when one is unable to predict future return distributions. A knowledge of the future return-risk-skewness structure of returns implies that security weights can be adjusted to improve the parameter structures of portfolios.

For the derivation of (3), see [25, Appendix A]. In essence, equation (2) can be decomposed into its skewness and coskewness components:

$$\sigma_n^3 = \sum_{i=1}^n x_i^3 \sigma_i^3 + \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n x_i x_j x_k \sigma_{ijk}$$

≠

where $\sum \sum \sum$ excludes those terms where $i=j=k$. Setting $x_i = \frac{1}{n}$ for all i and defining $\bar{\sigma}^3 = \frac{\sum_{i=1}^n \sigma_i^3}{n}$ and $\bar{\sigma}_{ijk} = \frac{\sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sigma_{ijk}}{n^3 - n}$, equation (3) is developed.

³See [25, Appendix C].

⁴See [25, Appendix D].

⁵For example, the t statistic for out of the money option cross-sectional skewness is 6.346, which is highly significant.

⁶From Elton and Gruber [9, page 420], the variance about portfolio expected return = $\frac{1}{n}(1 - \frac{n-1}{N-1})E(\bar{r}_i - \bar{r}_N)^2$. A comparison reveals that reductions in this variable are more than offset by reductions in cross-sectional skewness.

References

1. Fred D. Arditti, "Risk and the Required Return on Equity," Journal of Finance, Vol. 22 (March, 1967), pp. 19-36.
2. _____, "Another Look at Mutual Fund Performance," Journal of Financial and Quantitative Analysis, Vol. 6 (June, 1971), pp. 909-912.
3. W. L. Beedles, "On the Asymmetry of Market Returns," Journal of Financial and Quantitative Analysis, (September, 1979), pp. 653-660.
4. _____, "Return, Dispersion and Skewness," Journal of Financial Research, Vol 2 (Spring, 1979), pp. 71-80.
5. F. Black, "Fact and Fantasy in the Use of Options," Financial Analysts Journal, (July-August, 1975), pp. 36-41, 61-72.
6. F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, (May-June, 1973), pp. 637-654.
7. F. Black, M. Jensen and M. Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," in Studies in the Theory of Capital Markets, (Praeger; New York: 1972), pp. 79-121.
8. T. Conine and M. Tamarkin, "On Naive Diversification Given Asymmetry in Returns," forthcoming Journal of Finance (December, 1981).
9. E. Elton and M. Gruber, "Risk Reduction and Portfolio Size: An Analytical Solution," Journal of Business, (October, 1977), pp. 415-437.
10. E. Fama, Foundations of Finance, (Basic Books; New York: 1976), pp. 15-40.
11. E. Fama and J. MacBeth, "Risk, Return and Equilibrium: Empirical Tests," Journal of Political Economy, (May/June), 1973.
12. H. R. Fogler and R. Radcliff, "A Note on the Measurement of Skewness," Journal of Financial and Quantitative Analysis, Vol. 9 (June, 1974), pp. 485-489.
13. J. Francis, "Skewness and Investor Decisions," Journal of Financial and Quantitative Analysis, (March, 1975), pp. 163-172.
14. I. Friend and R. Westerfield, "Coskewness and Capital Asset Pricing," Journal of Finance, Vol. 35 (September, 1980), pp. 897-913.
15. G. Hawawini, "A Analytical Examination of the Intervaling Effect on Skewness and Other Moments," Journal of Financial and Quantitative Analysis, Vol. 15, No. 5, (December, 1980), pp. 1121-1127.

16. W. Jean, "The Extension of Portfolio Analysis to Three or More Parameters," Journal of Financial and Quantitative Analysis, Vol. 6 (January, 1971), pp. 505-515.
17. A. Krause and R. Litzenberger, "Skewness Preference and the Valuation of Risk Assets," Journal of Finance, Vol. 31 (September, 1976), pp. 1085-1100.
18. H. Levy, "A Utility Function Depending on the First Three Moments," Journal of Finance, Vol. 24 (September, 1969), pp. 715-719.
19. R. McEnally, "A Note on the Return Behavior of High Risk Common Stocks," Journal of Finance, Vol. 29 (May, 1974), pp. 199-202.
20. R. Merton, M. Scholes and M. Gladstein, "The Returns and Risk of Alternative Call Option Portfolio Investment Strategies," Journal of Business (April, 1978), pp. 183-242.
21. M. Miller and M. Scholes, "Rates of Return in Relation to Risk: Re-Examination of Some Recent Findings," Studies in the Theory of Capital Markets (Praeger; New York: 1972), pp. 47-78.
22. A. Mood, F. Graybill and D. Boes, Introduction to the Theory of Statistics, Third edition (McGraw-Hill; New York: 1974), pp. 180-181.
23. R. Reback, "Risk and Return on CBOE and AMEX Option Trading," Financial Analysts Journal, (July-August, 1975), pp. 42-52.
24. R. Roenfeldt, P. Cooley and M. Gombola, "Market Performance of the Options on the Chicago Board Options Exchange," Journal of Business Research, Vol. 1, (Number 1, 1979), pp. 95-107.
25. R. S. Sears, "Skewness, Diversification and Portfolio Performance: A Mathematical Analysis," Faculty Working Paper #809. University of Illinois at Urbana-Champaign, (September, 1981).
26. R. S. Sears and G. L. Trennepohl, "The Nature of Risk in Option Portfolios," Faculty Working Paper #767. University of Illinois at Urbana-Champaign, (April, 1981).
27. W. Sharpe, "A Simplified Model for Portfolio Analysis," Management Science, (January, 1963), pp. 277-293.
28. M. Simkowitz and W. Beedles, "Diversification in a Three-Moment World," Journal of Financial and Quantitative Analysis, (December, 1978), pp. 927-942.
29. G. Trennepohl and W. Dukes, "An Empirical Test of Option Writing and Buying Strategies Utilizing in-the-Money and Out-of-the-Money Contracts," Journal of Business, Finance and Accounting, Vol. 8, (Spring, 1981).



3 0112 060296198